LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **APRIL 2025**

UST 3502 – MATRIX AND LINEAR ALGEBRA

Date: 29-04-2025 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

	SECTION A - K1 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
1.	Define the following
a)	Hermitian matrix.
b)	Rank of a matrix.
c)	Vector Space.
d)	Eigen values and vectors.
e)	Quadratic forms.
2.	**Fill in the blanks**
a)	A matrix whose diagonal elements are the same and all other elements are zero is called ______________ matrix.
b)	The rank of a null matrix is ____________.
c)	The system of equations Ax = B is ______________ if rank of (A) = rank (A| B).
d)	____________________ theorem can be used in computing inverse of a matrix.
e)	______________ matrices have determinant equal to zero.
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**Match the following**
a)	The diagonal elements of a Skew symmetric matrix is — 2
b)	Rank of A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 1 & 2 \end{bmatrix}$ — Diagonal Matrix (D)
c)	A = $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ — I
d)	$AA' = A'A =$ — 0
e)	$P^{-1}AP$ — orthogonal matrix
4.	**True or False**
a)	|A| of a non-singular matrix is zero.
b)	$(AB)^T = A^T B^T$
c)	The number of linearly independent solutions of (A-λI) = 0 is equal to n - rank of |A-λI|.
d)	0 is a Characteristic root of a matrix iff the matrix is singular.
e)	The number of non - zero eigen values of the matrix A is called rank of the quadratic form.
	SECTION B - K3 (CO2)
	Answer any TWO of the following in 100 words each. **(2 x 10 = 20)**
5.	Solve the equation 2(x+B) = 3(x+A) + C where $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 & 2 \\ 3 & 2 & 5 \\ 1 & 2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 & 3 \\ 4 & 1 & -2 \\ 3 & 1 & 3 \end{bmatrix}$

6.	Solve the equations using cramer rule. $x + y + z = -1$ $x + 2y + 3z = -4$ $x + 3y + 4z = -6$
7.	Categorize the properties of linear dependence/independence.
8.	Obtain the matrices corresponding to the quadratic forms (i) $x^2 + 2y^2 + 3z^2 + 4xy + 5yz + 6zx$ $(ii)\ ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy$
	SECTION C – K4 (CO3)
	Answer any TWO of the following in 100 words each. (2 x 10 = 20)
9.	Categorize the properties of Determinants without proof.
10.	Examine for what values of λ, μ the system of equations $x + y + z = 6$ $x + 2y + 3z = 10$ $x + 2y + \lambda z = \mu$ have (i) no solution, (ii) a unique solution and (iii) an infinite number of solutions.
11.	Examine the properties of Characteristic roots & vectors.
12.	Discuss Homogenous and Non-Homogenous system of equations.
	SECTION D – K5 (CO4)
	Answer any ONE of the following in 250 words (1 x 20 = 20)
13.	Find the matrix where Y = AX represents a linear transformation for which $\begin{bmatrix}3\\1\\2\end{bmatrix} = A\begin{bmatrix}1\\0\\0\end{bmatrix}. \begin{bmatrix}2\\3\\2\end{bmatrix} = A\begin{bmatrix}0\\1\\0\end{bmatrix} \begin{bmatrix}3\\2\\1\end{bmatrix} = A\begin{bmatrix}0\\0\\1\end{bmatrix}$ Also find the images of the vectors a = $\begin{bmatrix}1\\1\\1\end{bmatrix}$, b = $\begin{bmatrix}4\\3\\7\end{bmatrix}$ and c = $\begin{bmatrix}5\\4\\0\end{bmatrix}$ under this transformation and show that (i) a and b are linearly independent. (ii) Aa and Ab are linearly independent (iii) a,b,c are linearly dependent.
14.	State and prove Cayley - Hamilton theorem.
	SECTION E – K6 (CO5)
	Answer any ONE of the following in 250 words (1 x 20 = 20)
15.	If A = $\begin{bmatrix}-1 & -2 & -2\\1 & 2 & 1\\-1 & -1 & 0\end{bmatrix}$, B = $\begin{bmatrix}-3 & -6 & 2\\2 & 4 & -1\\2 & 3 & 0\end{bmatrix}$, C = $\begin{bmatrix}-5 & -8 & 0\\3 & 5 & 0\\1 & 2 & -1\end{bmatrix}$ Prove that $(i)\ A^2 = B^2 = C^2 = 1$ (ii) AB = BA = C (iii) BC = CB = A (iv) AC = CA = B.
16.	Reduce the following symmetric matrix to a fundamental form and interpret the result of the quadratic forms : A = $\begin{bmatrix}6 & -2 & 2\\-2 & 3 & -1\\2 & -1 & 3\end{bmatrix}$

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